1. Alan Arnold Griffith and the "paper sheet theorem"

Slab avalanche release is controlled by destabilization of flaws located underneath the slab and on its flanks. Such destabilizations obey a very simple and quite useful criterion.

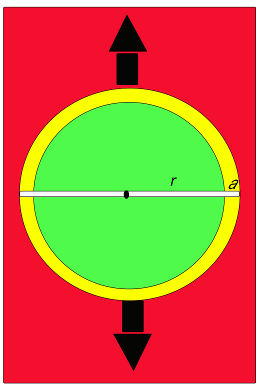
Stretching a rubber band requires energy. This energy is stored in the band in the form of so-called "elastic energy", which can be clearly evidenced when the applied tension is suddenly released (if the band fails for instance!).

Let us now stretch a paper sheet between hands. As for the rubber band, bonds between molecules elongate, but in a less extent, which is described by an increased stiffness. This mechanism also corresponds to some energy storage.

We shall now try to understand how the paper sheet may tear apart. Under constant tension, let us take a knife and cut a small notch perpendicular to the tension axis. Nothing happens. We slightly increase the notch size. Still nothing. And so on. At a given stage however, for a "critical" notch size, the sheet will suddenly tear apart, releasing the total stored energy.

The british engineer A. A. Griffith published in 1920 his famous criterion, that accounts for this phenomenon: under a tensile stress *σ*, the flaw critical size r^* obeys the equation $\sigma \sqrt{\pi r^{*}} = K_c$, where K_c is the so-called material toughness. This criterion shows that the larger the applied stress, the smaller is the critical flaw size, as intuitively expected.

This criterion can be explained in a very simple way, as follows. Let us consider a paper sheet loaded in tension, that stores elastic energy (in red in the figure below). Opening a crack perpendicular to the tensile axis needs some "tearing" energy in order to cut bonds between molecules, but doing so, it also relaxes part of the stored elastic energy, that helps the opening process. Roughly speaking, this last energy is taken from the green circular area of radius *r* on both sides of the crack. Increasing the crack size by a small amount *a* on both sides requires an additional tearing energy, partly balanced again by relaxation of the stored energy in the yellow crown. For the same crack size increase *2a* (*a* on both sides), the larger the crack, the larger will be the yellow belt area, and the corresponding relaxed energy used to help the cutting process. At a given point, this relaxed energy will be so large that it will overbalance the energy required to extend the notch further, which means that the knife becomes useless, and the notch size will increase spontaneously and dramatically.



This Griffith instability criterion is fundamental in all mechanical fracture processes, and particularly in avalanche release. It plays a key role in weak layer instability and crown crack opening, as detailed in the following notes.

François Louchet – data-avalanche.org